Part V:

(Solving System of Equation: Algebraically—by substitution)

Saturday Tutoring
Mathematics Program

Name:_____

8th Grade

Saturday Tutoring Program 8th Grade Mathematics Practice. Saturday, March 29, 2014.

Objective: SWBA to

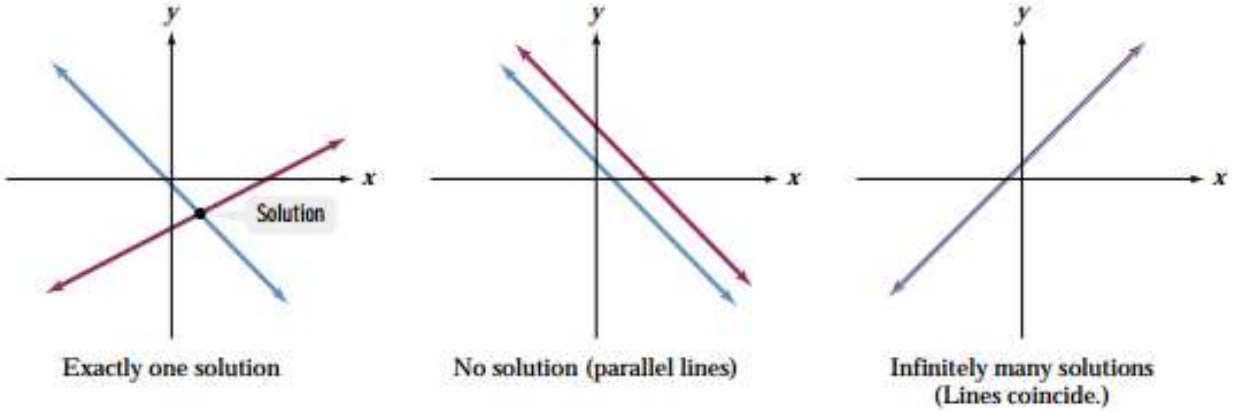
1. Analyze and solve pairs of simultaneous linear equations
2. Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. **(8.EE.8a, 8.EE8b)**

Intriduction

A **system of equations**, also known as **simultaneous equations**, is a set of equations that have multiple variables. The answer to a system of equations is a set of values that satisfies all equations in the system. Systems of equations can have multiple sets of answers that are correct. Last week we solved systems of equations graphically—that is by identifying the point where the lines intersected. Today, the solution of the system of equation algebraically—using the substitution method.

Remember the follow (this will help us speed up the work):

1. If the lines are parallel, that is their slope is the same, the system of equations has no solution because the lines never intersect.
2. The slopes of the lines of the system of equations are different, then there is one solution.
3. If the system of equations in made up of two identical lines—same slope and same y-intercept, then there is an infinite number of solutions.



Exactly one solution | No solution (parallel lines) | Infinitely many solutions (Lines coincide.)

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Mini-Lesson

To solve a system of linear equations algebraically, using the substitution method I follow these three steps:

THE SUBSTITUTION METHOD

STEP 1 Solve one of the equations for one of its variables.

STEP 2 Substitute the expression from Step 1 into the other equation and solve for the other variable.

STEP 3 Substitute the value from Step 2 into the revised equation from Step 1 and solve.

Let me demonstrate.

Solve the following system of equation:

$3x + 4y = -4$ **Equation 1**
$x + 2y = 2$ **Equation 2**

SOLUTION

1 Solve Equation 2 for x.

$x + 2y = 2$ **Write Equation 2.**

$x = -2y + 2$ **Revised Equation 2**

2 Substitute the expression for x into Equation 1 and solve for y.

$3x + 4y = -4$ **Write Equation 1.**

$3(-2y + 2) + 4y = -4$ **Substitute $-2y + 2$ for x.**

$y = 5$ **Solve for y.**

Using my knowledge of solving equations:

3(-2y+2) +4y=-4

-6y+6+4y=-4

-2y+6 = -4

-2y = -10

y=5

3 Substitute the value of y into revised Equation 2 and solve for x.

$x = -2y + 2$ **Write revised Equation 2.**

$x = -2(5) + 2$ **Substitute 5 for y.**

$x = -8$ **Simplify.**

▶ The solution is $(-8, 5)$.

✓**CHECK** Check the solution by substituting back into the original equations.

$3x + 4y = -4$	**Write original equations.**	$x + 2y = 2$
$3(-8) + 4(5) \stackrel{?}{=} -4$	**Substitute for x and y.**	$-8 + 2(5) \stackrel{?}{=} 2$
$-4 = -4$ ✓	**Solution checks.**	$2 = 2$ ✓

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Note: Use this online graphing calculator to graph equations (including systems of equations.)

http://go.hrw.com/math/midma/gradecontent/manipulatives/GraphCalc/graphCalc.html

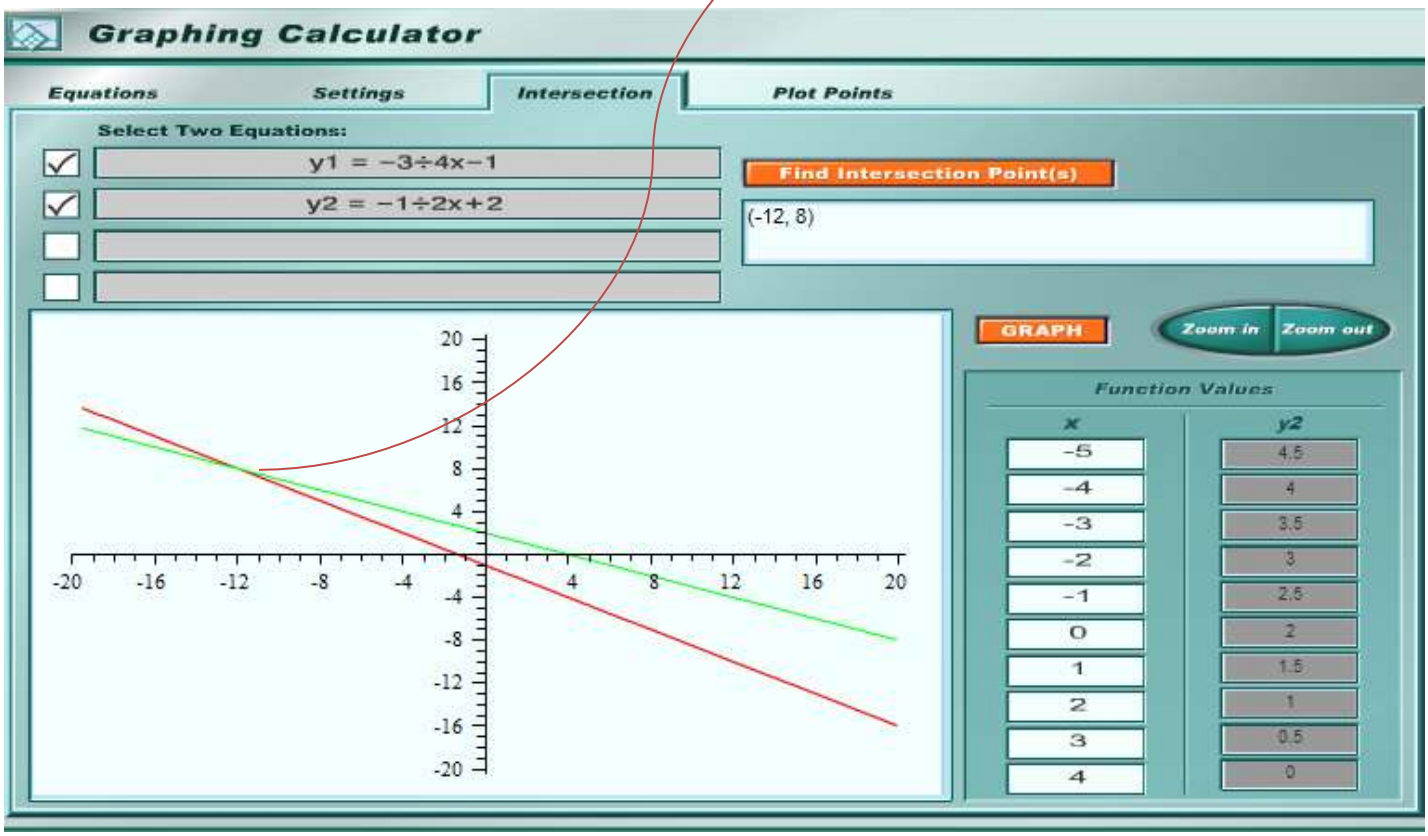
Let me verify the solution with a graphing calculator (see link above).

First, I have to write both equations in slope-intercept form before using the graphing calculator.

$3x + 4y = -4$ — Equation 1 → $y = \frac{-3}{4}x - 1$

$x + 2y = 2$ — Equation 2 → $y = \frac{-1}{2}x + 2$

Notice that the slopes the equations are different. Therefore, I can say for sure that these two lines are not parallel. As a result, these two lines will intersect at one point. That point must be (-8, 5)—the solution of the system of equations!



Guided Practice 1:

Solve the following system of equations:

$$2x - y = 7$$
$$6x - 3y = 14$$

Before we spend time solving algebraically let's analyze this system first by writing each equation in slope-intercept form and looking at the slope of the lines.

$$2x - y = 7 \text{ ------->} \; y = 2x - \frac{7}{2}$$

$$6x - 3y = 14 \text{ ------->} \; y = 2x - \frac{14}{6}$$

If we look carefully, we can see that the slope of each line is the same, m=2 (we do not care about the y-intercept, b). Therefore, we can conclude that these two lines are parallel. As a result, these two lines never intercept, and there is no solution!

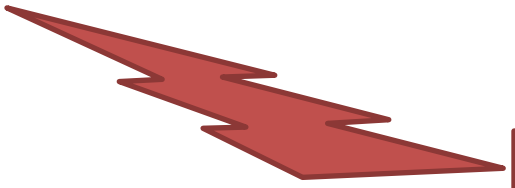
If we try to using substitution to solve for this system of equation, one the left side of the equation will not be the same as the right, rendering a mathematical impossibility!

Let's see. Let's substitute $2x - y = 7$ *or* $y = 2x - \frac{7}{2}$ into 6x-3y=14.

$$2x - \left(2x - \frac{7}{2}\right) = 7$$

$$2x - 2x + \frac{7}{2} = 7$$

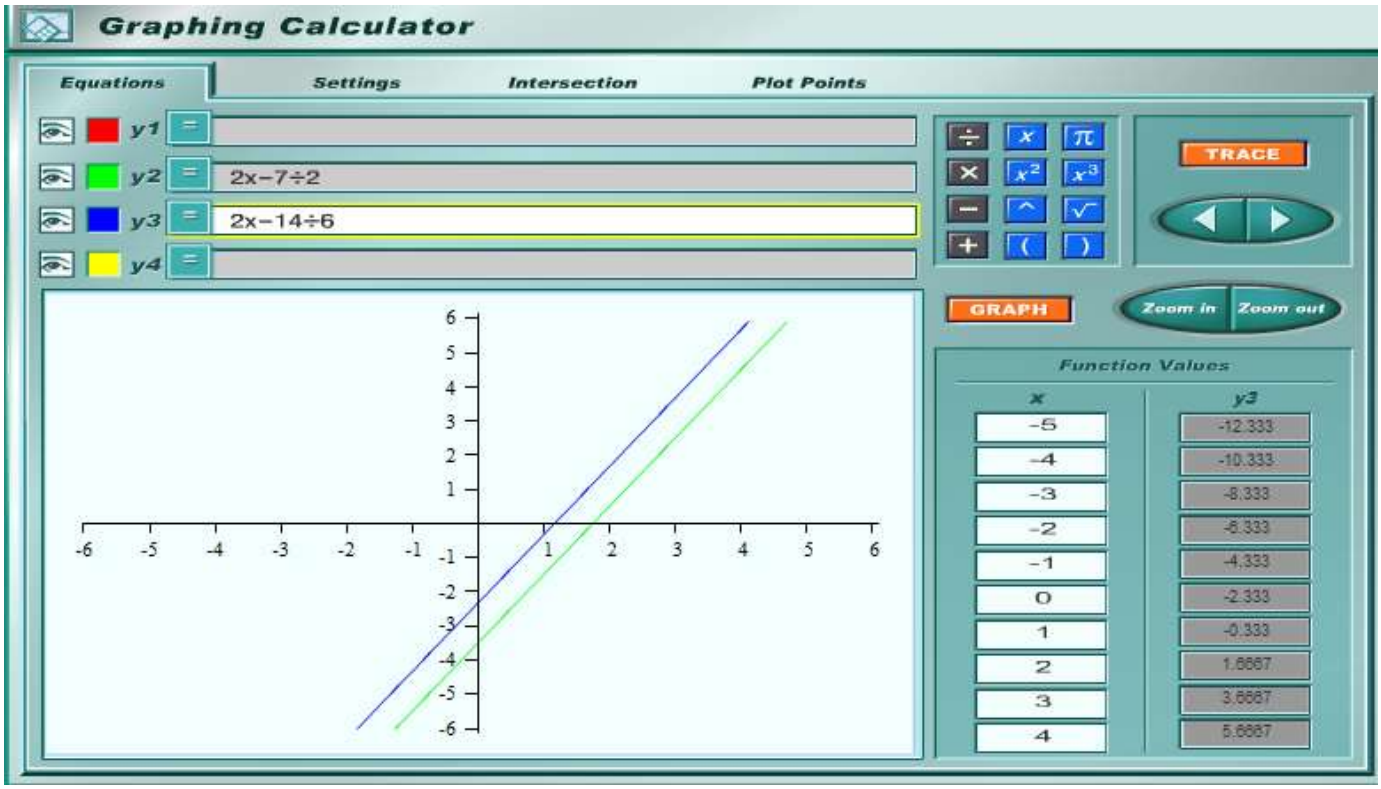
$$\frac{7}{2} = 7$$



This is impossible!

Therefore, there is no solution; which means that the lines are parallel!

Behold the power of mathematics! We can predict for sure that these lines will not touch. Let see the graph to collaborate this mathematical finding.



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Guided Practice 2:

Find the solution of this system of equations:

$$x + 3y = 2$$
$$4x + 12y = 8$$

Let us analyze this algebraically first. Let's re-write each equation in slope-intercept form.

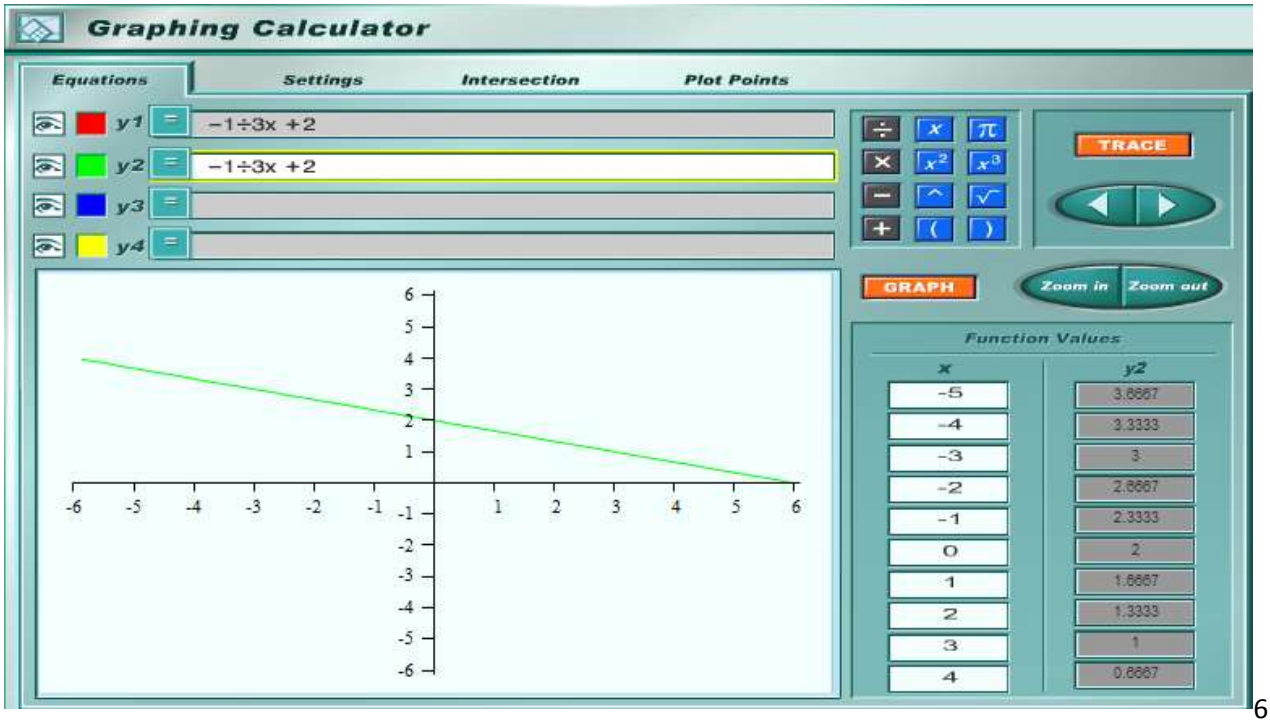
$$x+3y=2 \Longrightarrow y=-\frac{1}{3}x+2$$

$$4x+12y=8 \Longrightarrow y=-\frac{4x}{12}+\frac{8}{4} \Longrightarrow y=-\frac{1}{3}x+2$$

Holly macaroni! What do we have here?

The two equations represent two identical lines! This means that one line is superimposed onto the other. This implies that one of the lines touches intercept or touches each and every single point on the other line, but because lines are infinite, we have an infinite number of intersections, or an ***infinite number of solutions***!

This is how it would like if we graph this system of equations. There are actually two lines—one on top of the other. You will see one line blink once when you graph it.



Independent Practice:

Solve by substitution. Tell whether the system has *no solution, one solution* or *infinitely many solutions*. Then use the online graphing calculator to check your work.

1. $\begin{cases} y = x + 4 \\ y = 3x \end{cases}$

2. $\begin{cases} x = -2y + 1 \\ x = y - 5 \end{cases}$

3. $\begin{cases} y = 5x + 5 \\ y = 15x - 1 \end{cases}$

4. $\begin{cases} y = x - 7 \\ 2x + y = 8 \end{cases}$

5. $\begin{cases} y = 3x - 6 \\ -3x + y = -6 \end{cases}$

6. $\begin{cases} x + 2y = 200 \\ x = y + 50 \end{cases}$

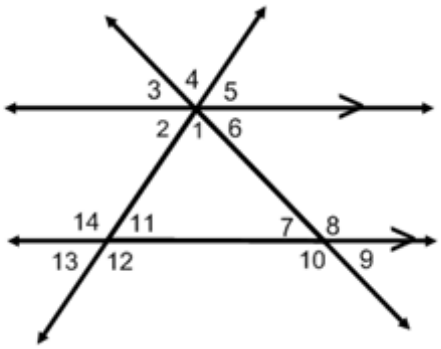
7. $\begin{cases} 2x + y = 3 \\ y = 2x + 1 \end{cases}$

8. $\begin{cases} y = \frac{3}{2}x \\ 6x - 4y = 1 \end{cases}$

Mixed Review:

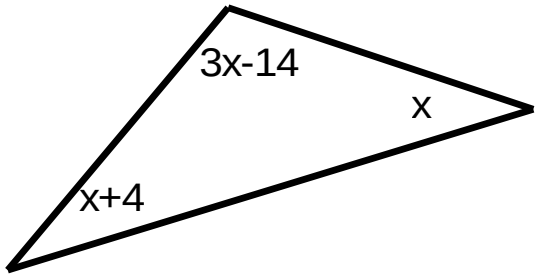
9. If the $m\angle 13 = 61°$ and $m\angle 9 = 46°$

Complete the table below for each angle and explain.



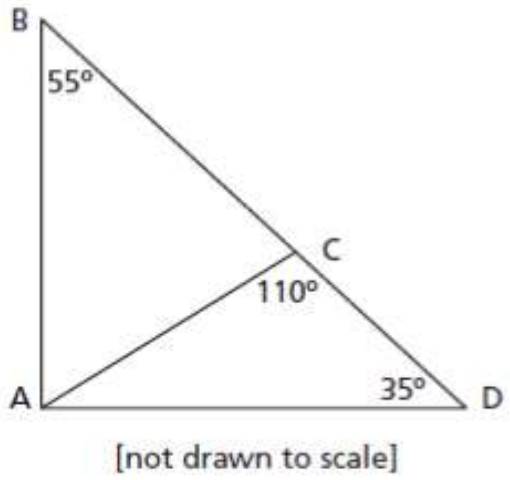
Angle	Measure	Reason
$\angle 11$		
$\angle 8$		
$\angle 2$		
$\angle 1$		
$\angle 7$		

10. Find the value of *x*. Find the degree measure of the angles.



Show your work.

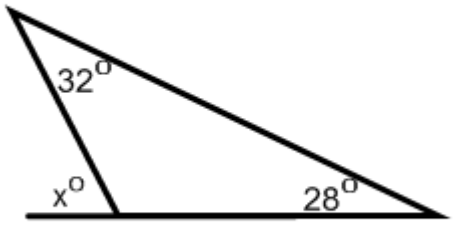
11. Fill in the missing angles in the diagram below:



[not drawn to scale]

Sow your work.

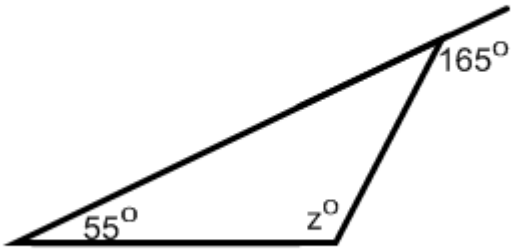
12. Find the value of x:



Show your work or explain

13. Find angle z.

Show your work or explain



14. Find x.