

Part IV:

*(Expressions and Equation—System of Equation:
Graphically and by substitution)*

**Saturday Tutoring
Mathematics Program**

Name: _____

8th Grade

Saturday Tutoring Program 8th Grade Mathematics Practice. Saturday, March 22, 2014.

Objective: SWBA to

1. Analyze and solve pairs of simultaneous linear equations
2. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.
3. Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. **(8.EE.8a, 8.EE8b)**

Introduction:

How do phone companies determine which plan to sell you? How do car rental companies or Taxi Company determine which rate to charge you? How do aerospace engineers determine when and where a rocket will land? If you start your own business, when can you expect to break even? Make a profit? Or have a loss?

The answer to all these questions involves the types of equations that we are currently working with in this class---Systems of Linear Equations or simultaneous equations. Systems of linear equations or simultaneous equations have many applications in science, engineering, business, sports, and many other areas, and they are often used as part of a decision-making process—to determine optimization and risk evaluation.

Systems of equations just mean that we are dealing with more than one equation and variable. So far, we've basically just deal with the equation for a line, which is $y = mx + b$. Now we going to use our knowledge solving equations and graphing equations in the form $y=mx + b$ find the solution or point of intersection or the breaking even-point of two lineal equations.

Mini-Lesson/Vocabulary:

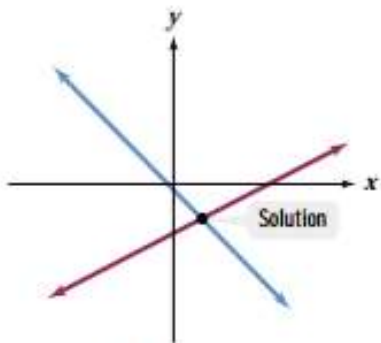
A **system of equations**, also known as **simultaneous equations**, is a set of equations that have multiple variables. The answer to a system of equations is a set of values that satisfies all equations in the system. Systems of equations can have multiple sets of answers that are correct. But today, we are going to work only with system of linear equations. The solution to a system of linear equation can be determined by graphing the two lines and seeing where they intersect. The solution is often written as ordered pairs, (x, y) and the coordinates of the point represent where

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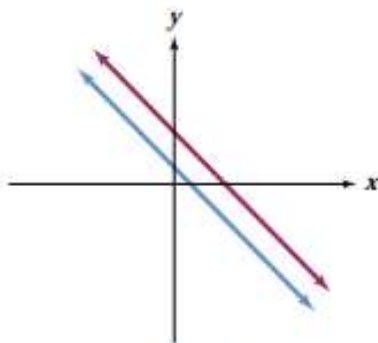
the lines intersect. There are other ways of solving system linear equations such as using the elimination and substitution methods.

Please notice that the following:

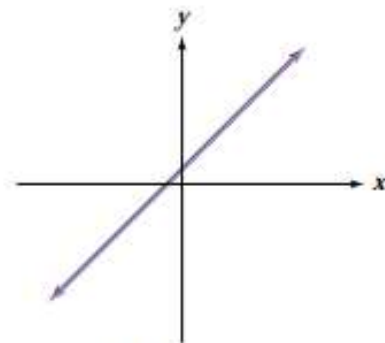
1. If the lines are parallel, that is their slope is the same, the system of equations has no solution because the lines never intersect.
2. The slopes of the lines of the system of equations are different, then there is one solution.
3. If the system of equations is made up of two identical lines—same slope and same y-intercept, then there is an infinite number of solutions.



Exactly one solution



No solution (parallel lines)



Infinitely many solutions
(Lines coincide.)

Let me demonstrate.

Find the solution of the following system of equations graphically.

$$y = x + 4$$

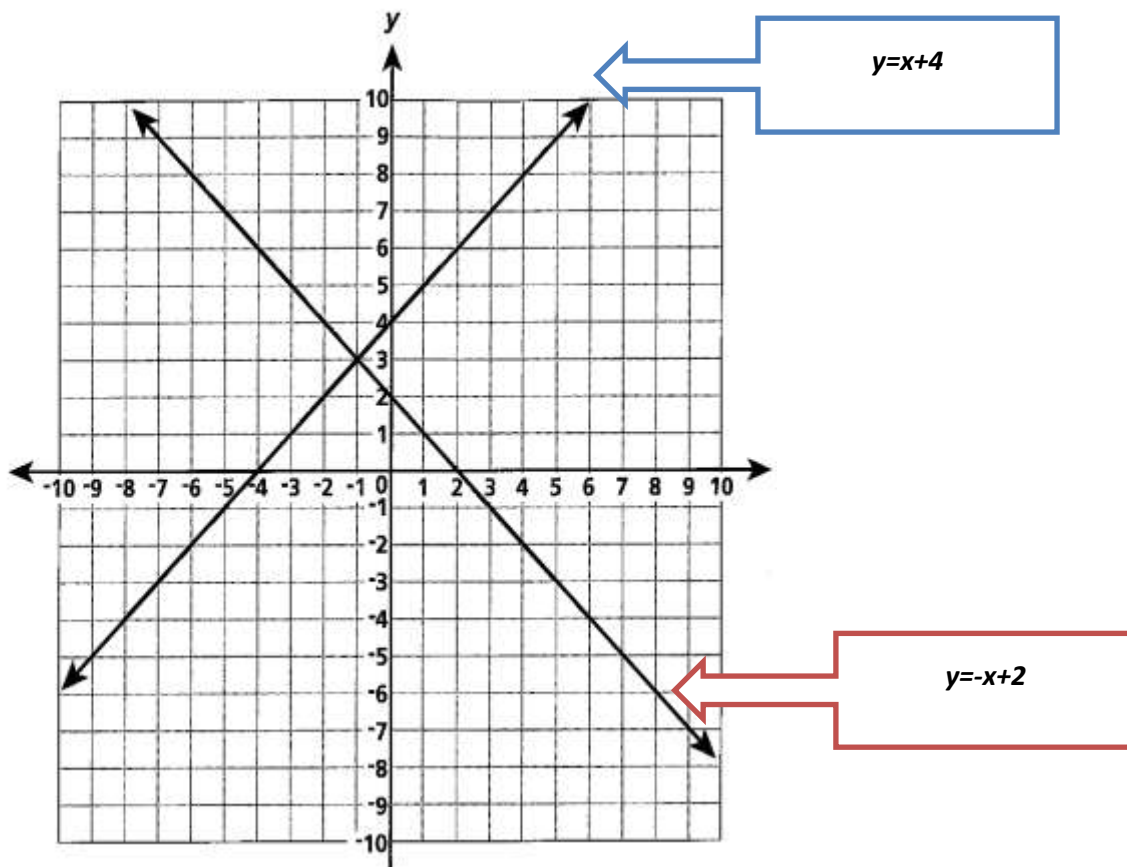
$$y = -x + 2$$

Step 1: re-write the equations in $y=mx+b$ form

(In this case the equations are already in slope-intercept form

Step 2: Graph each equation on an x-y plane.

I identify the slope and the y-intercept of each equation. In the first equation, $y=x+4$, the slope, $m=1 = 1/1$ and the y-intercept (b) = 4. In the second equation, $y=-x+2$, the slope, (m) = -1 = -1/1 and the y-intercept (b) = 2.



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Step 3: Find the point of intersection. The point of intersection of the two lines is (-1, 3)

Step 4: Check your answer.

If we substitute $x=-1$ and $y=3$ in each equation, they should be true.

Let's see.

Equation 1: $y=x+4$

$$3 = -1 + 4$$

$$3 = 3$$

Equation 2: $y= -x + 2$

$$3 = - (-1) + 2$$

$$3 = 1+2$$

$$3 = 3$$

Guided Practice 1:

A cell phone plan offers 300 free minutes for a flat fee of 20 dollars. If your usage exceeds 300 minutes, you pay 50 cents for each minute. A second cell phone plan offers 500 free minutes for a flat fee of 30 dollars. If your usage exceeds 400 minutes, you pay 30 cents for each minute

1. After using all the free minutes, at what point will the cost of the two phone plans be the same?
2. After using all the free minutes, when would the first company plan be better than the second company plan? And vice-versa?

Solution:

To answer these questions, and assume all free minutes for each plan have been used, we need to translate each company plan into algebraic equation of the form $y=mx + b$ and set up a system of equations. Remember, m represents the rate of change or slope, and b represents the initial cost or y -intercept, for each company plan.

Let x be the number of minutes you talk.

Let y be the cost.

$$Y_{\text{company 1}} = \$0.50x + \$20$$

or

$$y = \frac{1}{2}x + 20$$

$$Y_{\text{Company 2}} = \$0.30x + \$30$$

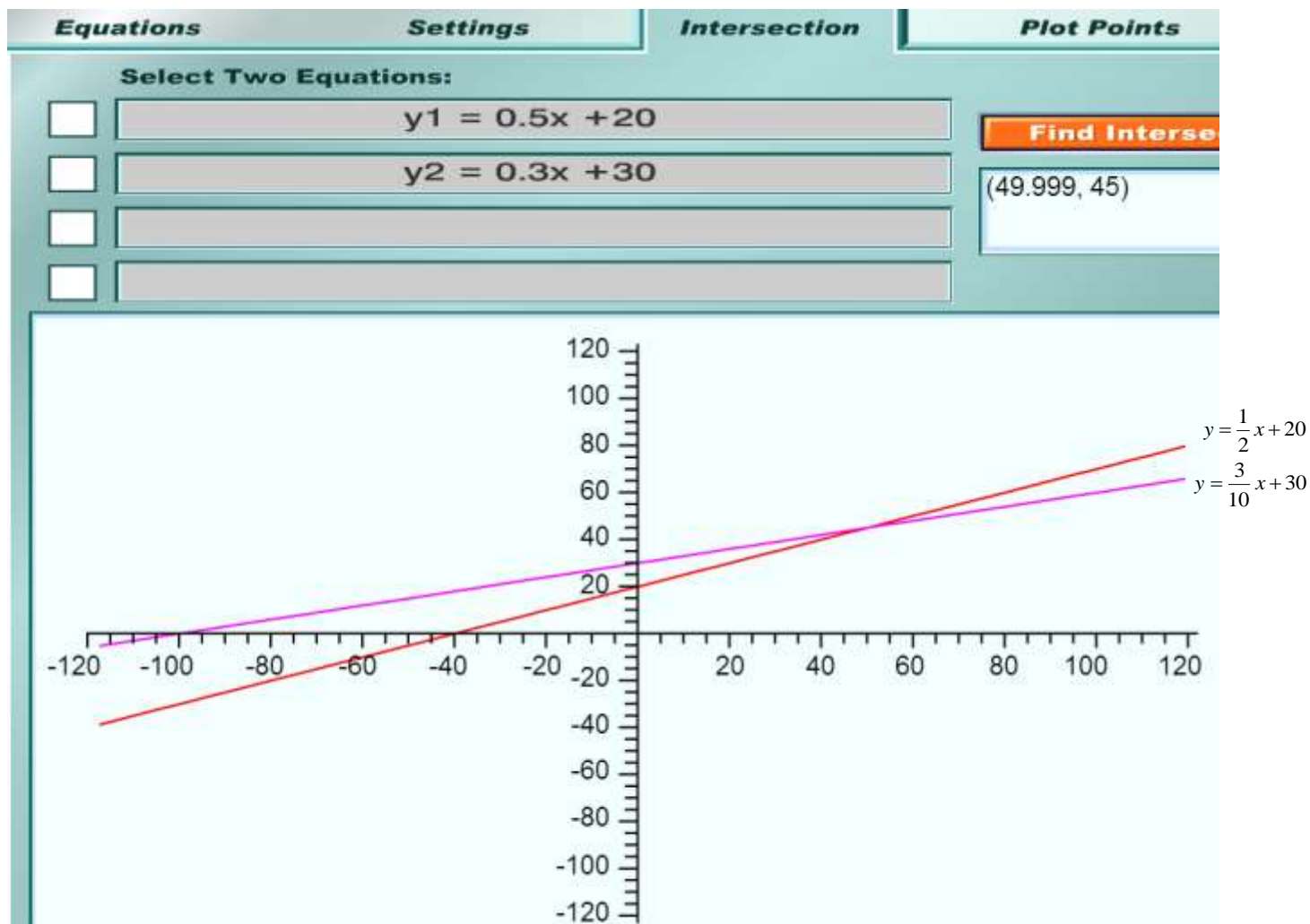
or

$$y = \frac{3}{10}x + 30$$

Rise

Run

We notice that both equations are written in slope-intercept form. Therefore, we graph the equations to find the point of intersection.



Note: Use this online graphing calculator to graph equations (including systems of equations.)

<http://go.hrw.com/math/midma/gradecontent/manipulatives/GraphCalc/graphCalc.html>

Guided Practice 2:

Solve the system of equations below.

$$\begin{cases} 5x + y = 4 \\ 2x - 3y = 5 \end{cases}$$

Step 1: re-write the equations in $y=mx+b$ form

(In this both equations have to re-written in slope-intercept form)

Let's rewrite the first equation: $5x + y = 4$

$$y = -5x + 4$$

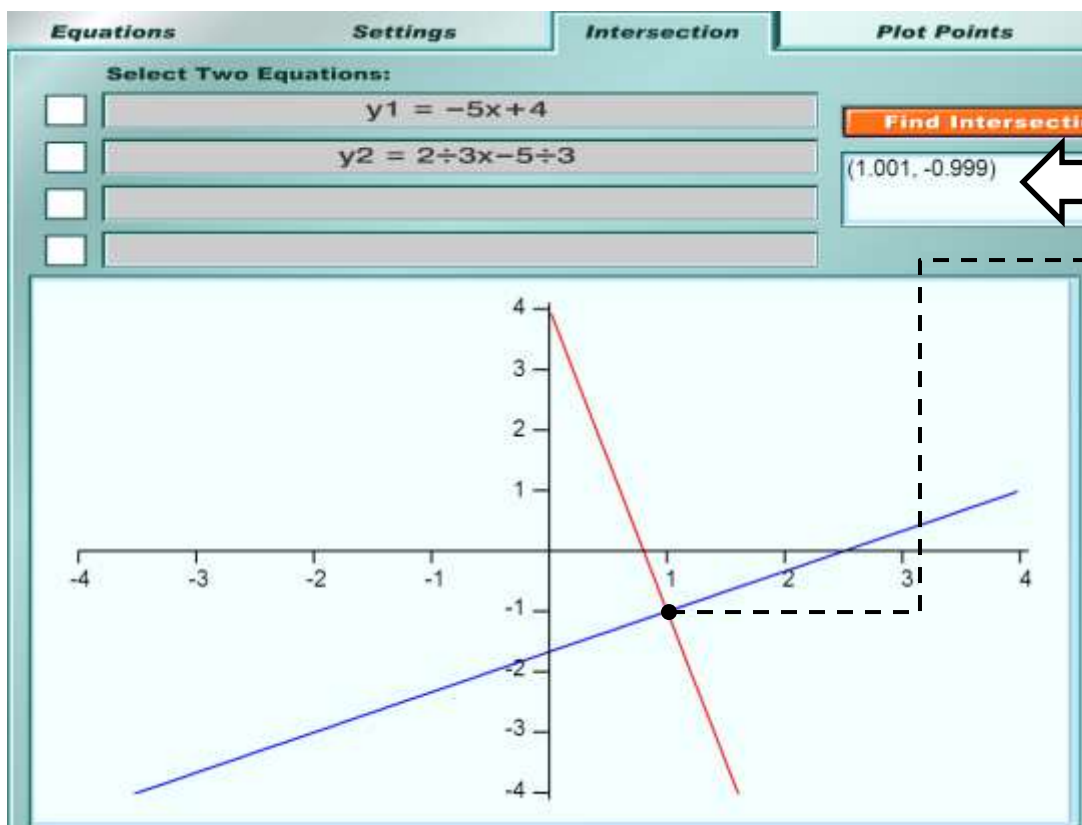
Let's re-write the second equation: $2x - 3y = 5$

$$-3y = -2x + 5$$

$$\frac{-3y}{-3} = \frac{-2x}{-3} + \frac{5}{-3}$$

$$y = \frac{2}{3}x - \frac{5}{3}$$

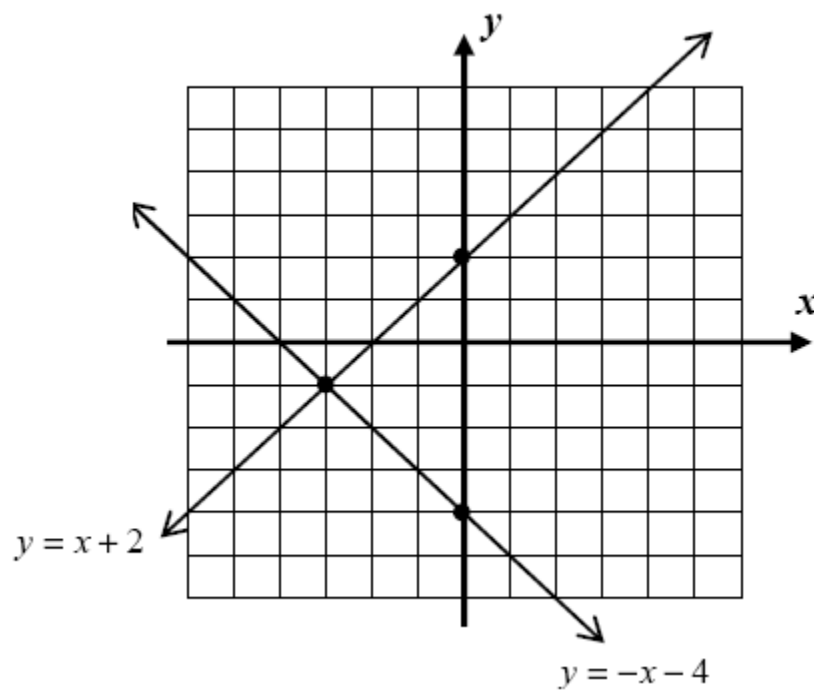
Step 2: Graph each equation on an x-y plane.



The coordinates of the point of intersection are $(1, -1)$

Independent Practice:

Problem 1: What is the solution of the system of equations graphed below?



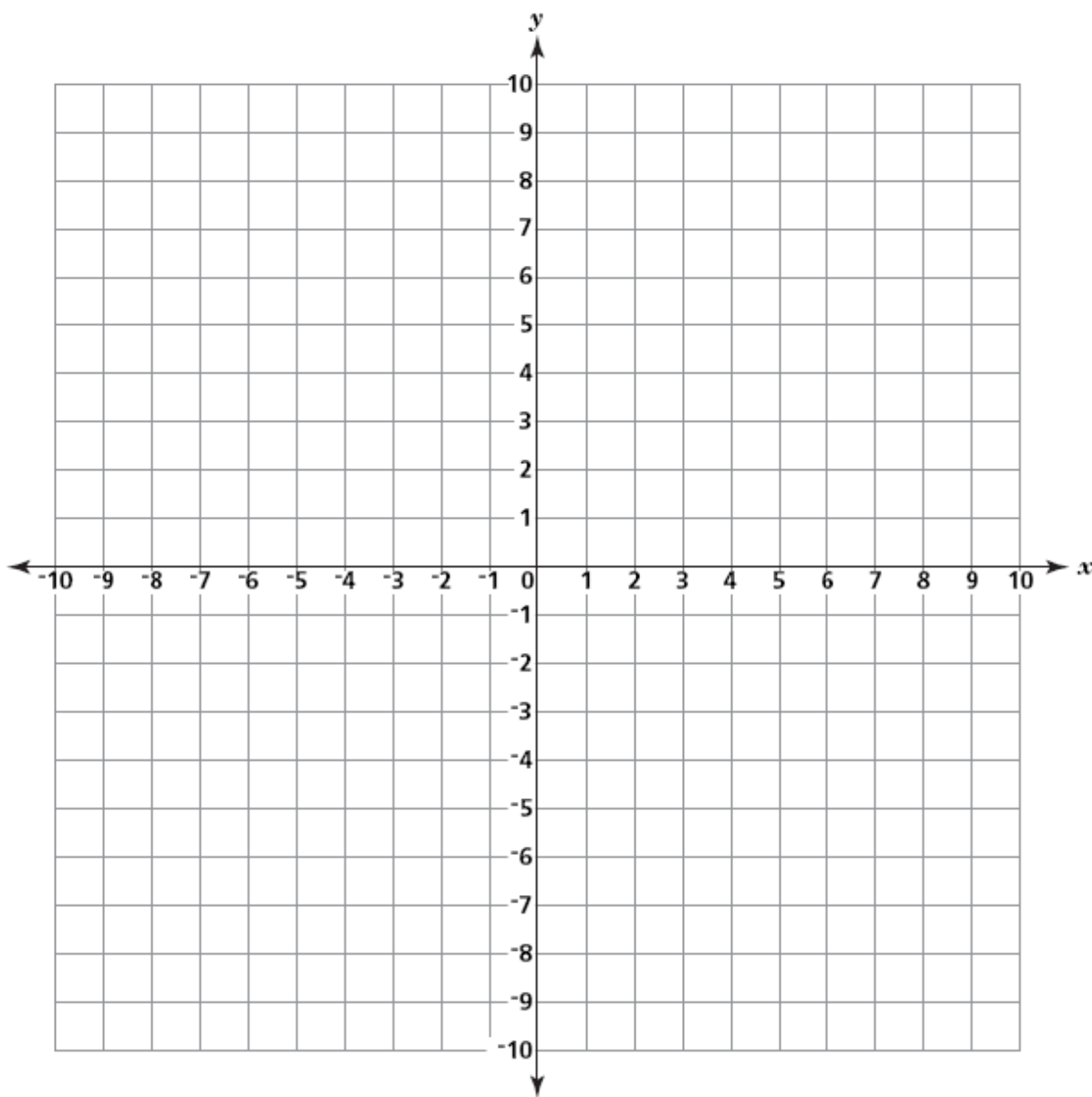
Answer: (,)

Saturday Tutoring Program 8th Grade Mathematics Practice. Saturday, March 22, 2014.

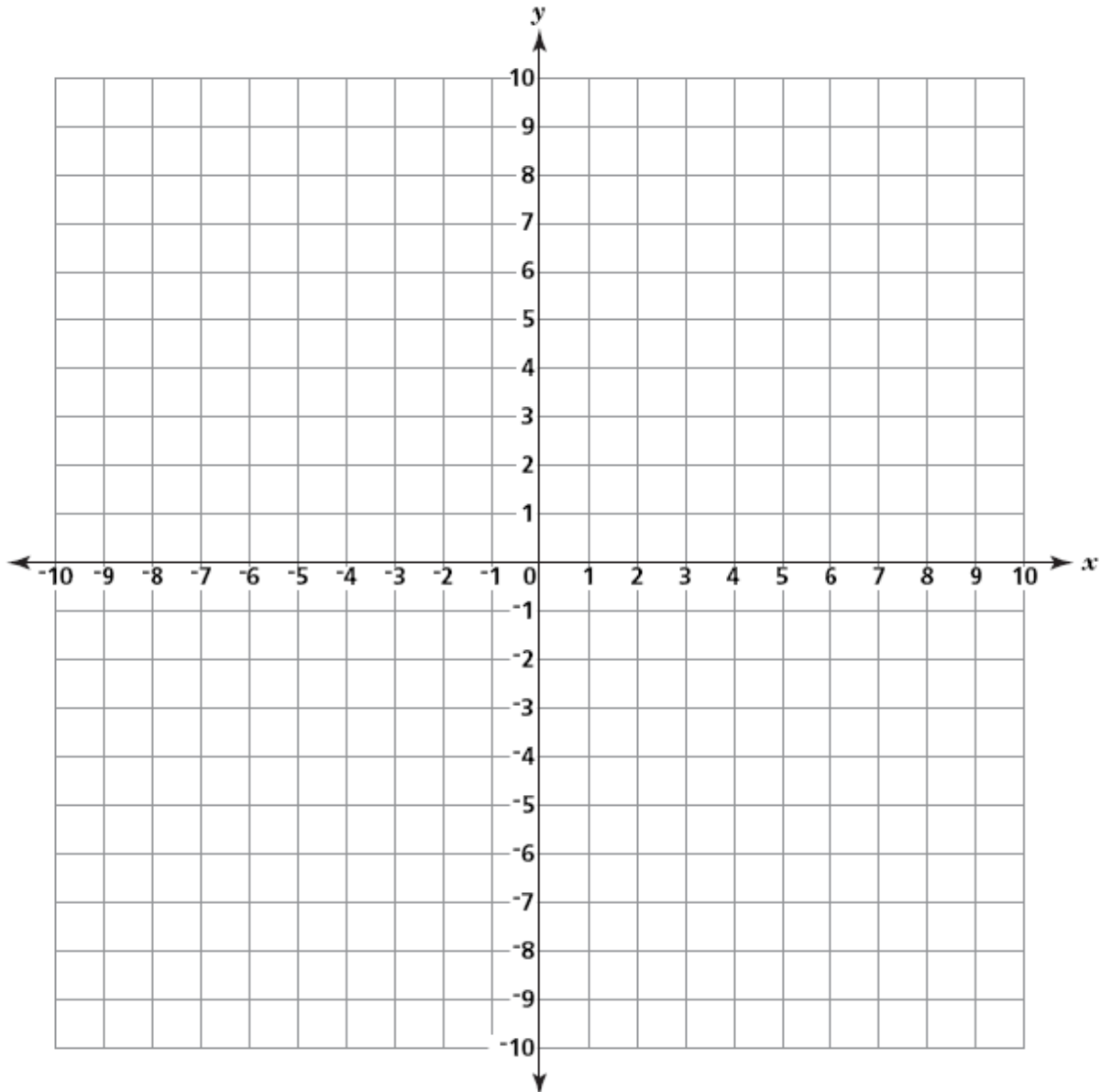
For each of the following questions, find the solution to the system of linear equations.

If there is *no solution* or *infinitely many*, explain why.

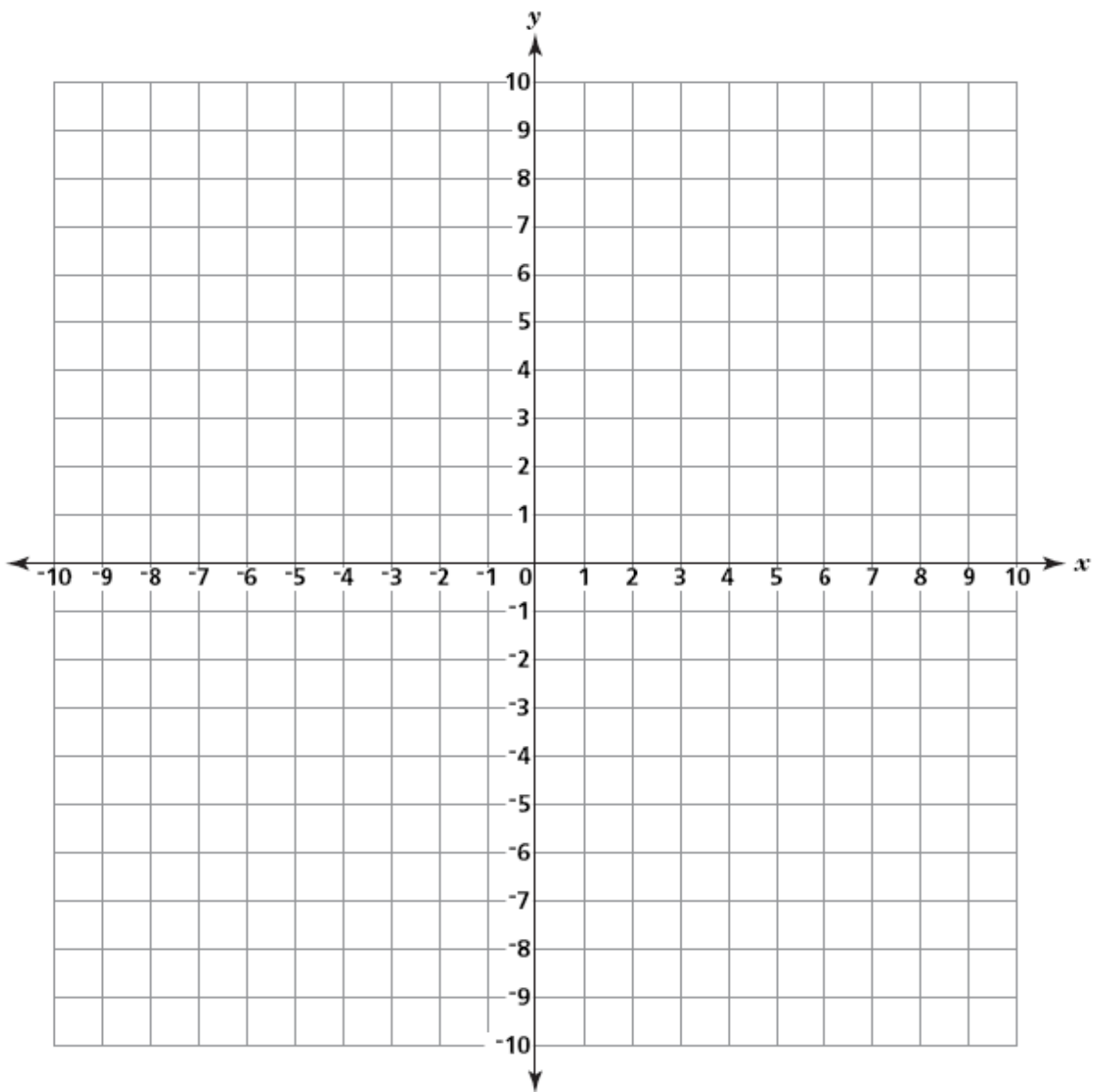
Problem 3:
$$\begin{cases} y = \frac{3}{5}x \\ y = \frac{3}{5}x - 2 \end{cases}$$



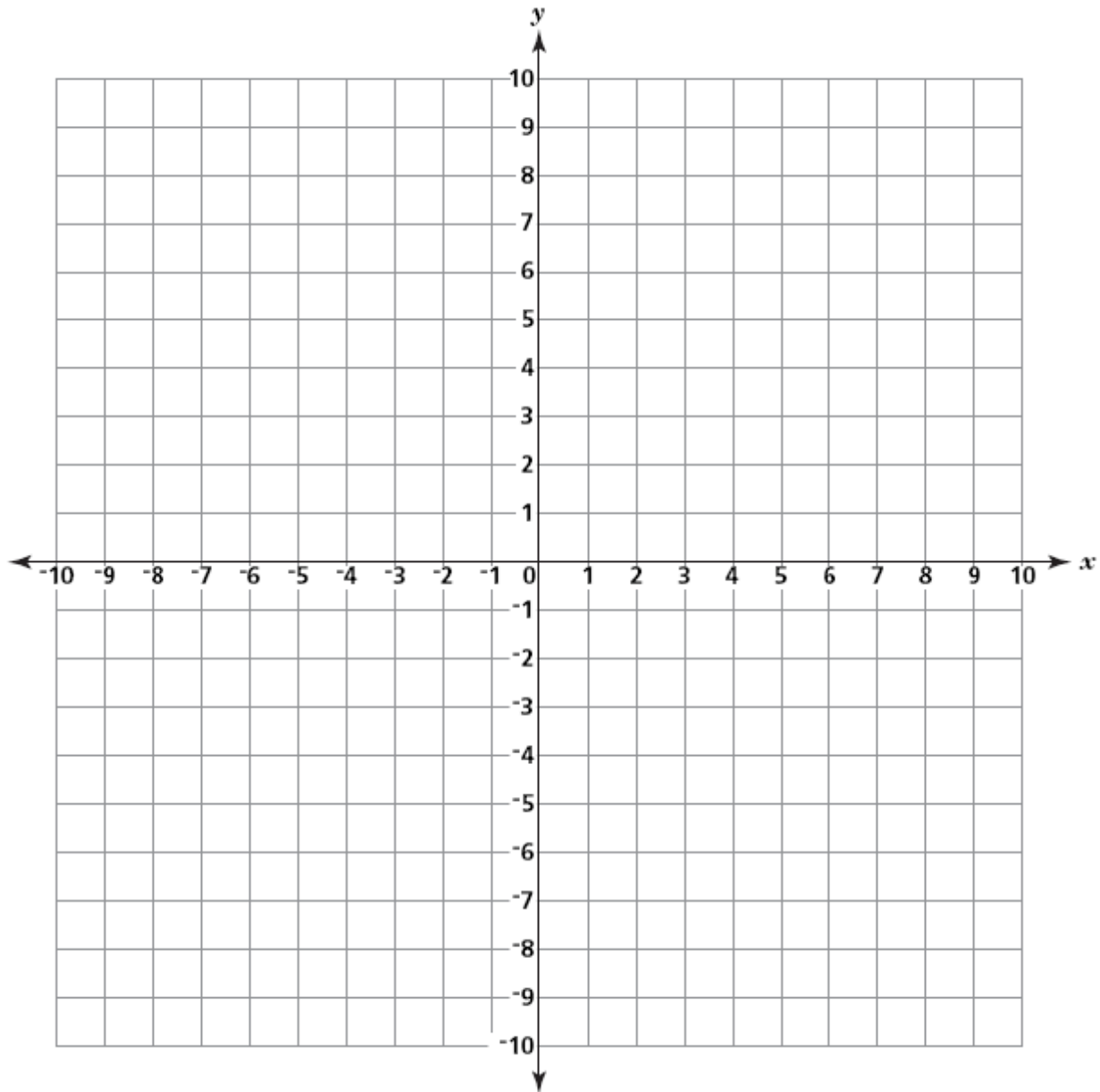
Problem 4:
$$\begin{cases} y = -\frac{1}{2}x + 2 \\ y = -x + 5 \end{cases}$$



Problem 5:
$$\begin{cases} -x + y = 2 \\ 4x - y = 1 \end{cases}$$



Problem 6:
$$\begin{cases} y = 4x + 10 \\ y = 3x + 9 \end{cases}$$



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Problem 7:

Sam needs to rent a car for a one-week trip to Florida. He is considering two companies:

A+ Auto Rental: \$175 plus \$0.10 per mile

Zippy Auto Rental: \$220 plus \$0.05 per mile.

Write and solve a system of equations to determine when the rental costs are the same for both companies. (Use a graphing calculator to graph the system of equations. Go to this website:

<http://go.hrw.com/math/midma/gradecontent/manipulatives/GraphCalc/graphCalc.html>

Show work here:

Problem 8: Find the solution of this system of lineal equations.

$$y = \frac{4}{3}x + 3$$

$$y = -\frac{2}{3}x - 3$$

